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Momentum–Impulse Balance and Parachute Inflation: Rocket-Propelled Payloads

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I. Introduction

THIS Note is the fifth and last of a series dedicated to the analysis of parachute inflation using the momentum-impulse (MI) theorem [1–4]. Much insight has been gained, in particular, with regards to specific examples such as parachute drops from fixed points [2], parachute cluster inflation [3], and parachute disreefing [4]. This Note shall focus on a lesser-known application, namely that of parachutes inflating while being attached to payloads generating thrust of their own. Examples include drogue parachutes inflating behind ejection seats that are being rocketed away from disabled aircraft, or braking chutes deployed behind aircraft whose engine is being throttled up. The use of the MI theorem shall yield new results that incorporate explicitly the effects caused by the impulses contributed by gravity and by thrust. Some conceptual clarification will be discussed as well, in relation to the mass ratio concept. This will result in a prescription that defines how to use the opening shock data of free-falling payloads [5–7] for the study of "motored" inflation.

II. Acceleration Profiles: Motored- vs Gravity-Powered Inflation

As a general rule, the maximum drag force $F_{\max}$ generated by a parachute during its inflation depends partly on how much kinetic energy the parachute-payload system has at its disposal at the beginning of the inflation process. But $F_{\max}$ should depend also on how much energy is being injected into the system during inflation, by an outside source such as gravity [1–3] or a thrust force produced by rockets, jet turbines, or (motored) propellers attached to the parachute's payload. Superficially, motored inflation appears similar to inflation fueled by gravity only. But it is different at a deeper level, in particular, with respect to payload-mass scaling and overall dependence on mass ratio.

The drag force generated by any bluff body moving through a fluid is a direct reflection of the type of wake that it is generating. Therefore, the drag of an accelerating body shall be different from that of a decelerating body [8–17]. This difference is a direct result of the wake stretching and falling further behind an accelerating body, in contrast with the wake "catching up" with and "pushing" a decelerating body [15–17]. For this reason much insight on $F_{\max}$ can be gained when inflation events are classified in terms of a parameter that can be directly related to motion evolution profiles. This has been achieved already in the case of gravity-powered inflation, with the use of the "inverse" mass ratio R_m [1,5–7]. Defined as $R_m \equiv \rho(SC_D)_{\mathrm{sd}}^{3/2}/m$, the mass ratio involves the a priori knowledge of the atmospheric density ρ at deployment altitude and of the total mass m of the parachute-payload system. The parachute steady-descent drag area $(SC_D)_{\mathrm{sd}}$ is calculated from the standard descent rate formula, namely, $(SC_D)_{\mathrm{sd}} = 2\ \mathrm{mg}/\rho V_{\mathrm{sd}}^2$. The value of the mass ratio will be correlated with motion profiles in the following way: in regimes characterized by $R_m \to \infty$, a parachute-payload system will tend to decelerate substantially during inflation, as the mass of the air that is comoving with the parachute is much larger than the payload mass, thereby generating a large amount of drag compared to weight. In contrast, systems characterized by $R_m \to 0$ involve much smaller air masses and drag forces and will typically move at constant speed along horizontal trajectories, or accelerate at a rate mostly controlled by the value of the gravitational acceleration constant g along near-vertical trajectories. Indeed, inflation data such as $F_{\max}$ have been shown to vary substantially according to whether R_m is small or large [1,5–7].

Unfortunately, such a correspondence no longer holds if the payload is powered by additional external forces. For example, there will be rocket-powered systems that exhibit characteristics that are the exact opposite of those gravity-only cases. One can consider high-R_m systems, such as light payloads hanging under "large" parachutes, that will sustain accelerations and high inflation loads when equipped with strong-enough pusher-type rockets. A similar opposite trend appears with low-R_m systems, for example, when small parachutes carrying heavy payloads are equipped with retrorockets generating thrust that yields substantial deceleration and decrease in drag force. It is therefore clear that an alternative to the mass ratio R_m must be found. The MI theorem is used to good effect in the next section to do just that.

III. MI Theorem and Effective Mass Ratio

The MI theorem relates the momentum change of a parachute-payload system to the time integral of the net external forces acting on it. Focusing on the momentum and force components along the fall trajectory of an inflating parachute, one has the following expression, which involves the time-varying parachute drag force $F_D(t)$ and additional force $F_R(t)$, and the parachute-payload weight component tangential to the trajectory:

$$mV_f - mV_i = \int_i^f F_D(t)\,\mathrm{d}t + \int_i^f F_R(t)\,\mathrm{d}t + \int_i^f W\cos\theta(t)\,\mathrm{d}t \quad (1)$$

Here the flight angle $\theta(t)$ is defined so that a 0 deg value corresponds to a parachute-payload system falling straight down, and a 90 deg value to a system traveling horizontally (and to the right). As in [1], $V(t_i) \equiv V_i$ and $V(t_f) \equiv V_f$, and refer to the parachute-payload tangential speeds at the beginning and end of the inflation process, respectively. Note that with this notation, both V_i and V_f are positive as long as $0 < \theta(t) < 90$ deg [This discussion will not include the cases where $\theta(t) > 90$ deg, that is, systems traveling upward. These can be studied using the same approach, although some definitions

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will need to be revisited whenever V_i becomes negative.] Drag being the opposite to motion means that $F_D(t) < 0$ and $F_{\max} > 0$.

As with [1], Eq. (1) is rewritten in terms of the maximum parachute drag force $F_{\max}$ and of its nondimensional rendition, the opening shock factor C_k:

$$(mV_f - mV_i) = -\left(\frac{1}{2}\rho C_k (SC_D)_{\mathrm{sd}} V_i^2 (t_f - t_i)\right) I_F^{if} + \langle F_R\rangle (t_f - t_i) + \int_i^f W\cos\theta(t)\,\mathrm{d}t \tag{2}$$

with C_k being defined by

$$F_{\max} \equiv \tfrac{1}{2}\rho C_k (SC_D)_{\mathrm{sd}} V_i^2 \tag{3}$$

Herein the nongravitational external force will be referred to as "rocket thrust," and will be formulated in terms of the averaged thrust constant $\langle F_R\rangle$ defined as follows:

$$\langle F_R\rangle = \pm \int_i^f \frac{|F_R(t)|\,\mathrm{d}t}{(t_f - t_i)} \tag{4}$$

As in [1], the so-called *drag integral* is defined as

$$I_F^{if} = \int_i^f \frac{|F_D(t)|\,\mathrm{d}t}{F_{\max}(t_f - t_i)} \tag{5}$$

The drag integral I_F^{if} in Eq. (5) contains the information related to the normalized shape of the F_D-vs-t curve and is a measurement of the area under that curve [1]. Note that Eqs. (2) and (5) are valid as long as $F_D < 0$ throughout inflation. Note also that the $\pm$ symbol in Eq. (4) means assigning a "-1" factor when $F_R(t)$ points in the same direction as drag, that is, when the thrust force is in a "retrorocket" configuration; and a "$+1$" factor is assigned whenever F_R is antiparallel to F_D, that is, whenever in a "pusher-rocket" configuration. In this model the value of F_R is assumed to always be tangential to the trajectory, as well as to *never* change orientation during the inflation process [this last part is implicit in Eq. (4)]. Finally, F_R will correspond to the actual amount of rocket thrust that exceeds the drag of the payload container/body.

At this point several dimensional and nondimensional numbers must be defined.

1) The effective mass m_{eff}:

$$m_{\mathrm{eff}} \equiv m + \frac{\langle F_R\rangle}{g} \tag{6}$$

2) The effective mass ratio R_m^{eff}:

$$R_m^{\mathrm{eff}} \equiv \frac{\rho (SC_D)_{\mathrm{sd}}^{3/2}}{m_{\mathrm{eff}}} \tag{7}$$

3) The filling time n_{fill} and generalized filling time $n_{\mathrm{fill}}^{\mathrm{gen}}$ (same definitions as in [1]):

$$n_{\mathrm{fill}}^{\mathrm{gen}} \equiv \frac{V_i(t_f - t_i)}{(SC_D)_{\mathrm{sd}}^{1/2}} I_F^{if} \equiv n_{\mathrm{fill}} \frac{D_0}{(SC_D)_{\mathrm{sd}}^{1/2}} I_F^{if} \tag{8}$$

Here D_0 is the *nominal* canopy diameter, defined from the total canopy surface area S_0 as $D_0 = (4S_0/\pi)^{1/2}$ for hemispherical-type canopies [5].

Equation (2) can now be solved for C_k, yielding

$$C_k = \frac{2}{R_m^{\mathrm{eff}} n_{\mathrm{fill}}^{\mathrm{gen}}} \Gamma_{\mathrm{eff}} \tag{9}$$

where

$$\Gamma_{\mathrm{eff}} \equiv \left(1 - \frac{V_f}{V_i}\right) + \left(\frac{V_f}{V_i} - 1\right)\frac{\langle F_R\rangle}{m_{\mathrm{eff}} g} + n_{\mathrm{fill}}\left[\frac{\langle F_R\rangle}{m_{\mathrm{eff}}}\frac{D_0}{(V_i)^2}\right] + n_{\mathrm{fill}}\left(\frac{\int_i^f \cos\theta(t)\,\mathrm{d}t}{t_f - t_i}\right)\frac{m}{m_{\mathrm{eff}}}\frac{gD_0}{(V_i)^2} \tag{10}$$

Equation (10) represents, in units of effective mass (m_{eff}) and initial speed, the momentum lost by the system, as well as the gravitational and "rocket" impulses sustained. Note that without rocket power, that is, $\langle F_R\rangle = 0$, the value of the effective mass ratio and opening shock factor reduce to the mass ratio (R_m) and opening shock factor used in [1]. In other words, the results derived for gravity-powered inflation are special cases of Eqs. (9) and (10). The fourth term in Eq. (10) represents the usual gravitational impulse that arises along nonhorizontal trajectories. It is written here as a "Froude" term, that is, expressed in terms of the well-known Froude number $F_r = V_i^2/gD_0$ [1]. The third term, being proportional to $\langle F_R\rangle D_0/V_i^2$, represents the total impulse provided by the rocket. It is expressed in a form similar to the gravitational impulse term, that is, with a Froude-like factor that uses an effective acceleration constant (i.e., $\langle F_R\rangle/m_{\mathrm{eff}}$). Last, the second term in $V_f/V_i - 1$ is an adjustment to the momentum change that results from the characterization of the mass ratio in terms of an effective mass (i.e., m_{eff}) instead of the system's real mass (i.e., m).

Using Eqs. (6) and (7) to define an effective mass and mass ratio is obviously not unique. But their merit lies in yielding, together with Eqs. (2–4), a formula for C_k that is identical in form to that of the gravity-only case [1]. Moreover, using R_m^{eff} as defined in Eq. (7) appears to yield the desired motion profile classification parameter. Indeed:

In the pusher-rocket configuration, the limit $\langle F_R\rangle \to \infty$ yields $R_m^{\mathrm{eff}} \to 0$ while at the same time corresponding to an accelerated motion at all values of the parachute-payload mass m. On the other hand, the limit $\langle F_R\rangle \to 0$ yields $R_m^{\mathrm{eff}} \to 0$ only when $m \to \infty$ (acceleration or constant speed), and $R_m^{\mathrm{eff}} \to \infty$ only when $m \to 0$ (deceleration), as in the gravity-only case. Note that Γ_{eff} has the same limits as in the gravity-only case as well, namely, $\Gamma_{\mathrm{eff}} \to 1$ as $R_m^{\mathrm{eff}} \to \infty$ and $\Gamma_{\mathrm{eff}} \to 0$ as $R_m^{\mathrm{eff}} \to 0$.

In the retrorocket configuration, *and* within the range $\langle F_R\rangle/g = [-m, 0]$, one is mainly in the high effective mass ratio range (i.e., large-R_m^{eff}) in which deceleration profiles are the rule of the day. Note, in the range $\langle F_R\rangle/g = [-\infty, -m]$ the system will transition from a retrorocket configuration to a pusher-rocket configuration, as the motion of the parachute-payload reverses directions. Such a scenario lies outside the scope of this formulation.

It is interesting to note that in most cases where $\langle F_R\rangle \neq 0$, the values of $n_{\mathrm{fill}}^{\mathrm{gen}}$ and Γ_{eff} at a given R_m^{eff} should be similar to those values of $n_{\mathrm{fill}}^{\mathrm{gen}}$ and Γ seen with the gravity-only cases at $R_m = R_m^{\mathrm{eff}}$, being characterized by the same acceleration/deceleration profile. At least this is what can be expected, as long as the rocket causes the parachute to generate a wake that is typical of gravity-only cases—for example, rockets with a constant impulse. Under this constraint, one should expect the C_k vs R_m^{eff} graphs in the $\langle F_R\rangle \neq 0$ regime to be similar to the C_k vs R_m graphs obtained in the gravity-only cases [1,5–7]. In other words, gravity-only parachute data could be used to estimate the C_k values of rocket-powered payloads. For example, personnel-type parachutes of diameters approximating $D_0 \sim 30$ ft and carrying 250 lb payloads should have the same opening shock factor value (i.e., $C_k \sim 0.1$), as when carrying a 450 lb payload that is equipped with a retrorocket generating a constant 200 lb thrust, given that $R_m|_{250} = R_m^{\mathrm{eff}}|_{450} \sim 3$ (assuming similar filling times n_{fill} [1]). Similarly, one can expect to find the opening shock factor at $C_k \sim 0.4$ when the same chutes are carrying 250 lb payloads and propelled by a pusher rocket generating in excess of 2000 lbs, as here $R_m^{\mathrm{eff}}|_{450} \sim 0.4$.

IV. Simple Maximum Drag Formulas

As with the other papers of this series [1–4], several useful formulas for $F_{\max}$ itself can be derived from Eqs. (3), (9), and (10). For inflation events taking place along the horizontal the value of the flight angle remains at $\theta(t) = 90$ deg, a case for which Eqs. (9) and (10) yield

$$F_{\max} \equiv \frac{1}{2}\rho(SC_D)_{\mathrm{sd}}V_i^2\left(\frac{2}{R_m^{\mathrm{eff}} n_{\mathrm{fill}}^{\mathrm{gen}}}\right) \times \left\{1 - \frac{V_f}{V_i} + \left(\frac{V_f}{V_i} - 1\right)\frac{\langle F_R\rangle}{m_{\mathrm{eff}}g} + n_{\mathrm{fill}}\left[\frac{\langle F_R\rangle}{m_{\mathrm{eff}}g}\frac{gD_0}{(V_i)^2}\right]\right\} \tag{11}$$

Although gravity is not a factor for this type of trajectory, the factor of g found here has its origin in the factor $\langle F_R\rangle/g$ that is used as a mass in the definition of m_{eff}. This definition amounts to regarding rocket force as being equivalent to a weight being added to or removed from the payload when falling vertically. Note also that the constraint $F_{\max} > 0$ can always be used to check the soundness of the various approximations that can be obtained from Eq. (11). In particular, the constraint can yield upper bounds on the value of n_{fill}, which may be a useful feature in cases where n_{fill} has not been well documented experimentally.

One interesting approximation of Eq. (11) arises when a pusher rocket is generating thrust in amounts that are much greater than mg. Because $\langle F_R\rangle/m_{\mathrm{eff}}g \sim 1$, the two terms in $1 - V_f/V_i$ will nearly cancel each another. Being left with the term in n_{fill} means that

$$F_{\max} \sim \left(\frac{\langle F_R\rangle}{I_F^{if}}\right) \qquad \text{Horizontal inflation} \tag{12}$$

This simple expression depends on the drag integral I_F^{if} which, as explained in [1], is in the range $I_F^{if} \sim 1/4 - 1/2$ and is a result of the actual shape of the $F_D(t)$ versus t curve. In turns, such a shape depends implicitly on m, V_i, D_0, (SC_D), ρ, etc. Of course, Eq. (12) points to the fact that in this case rocket thrust will generate very large opening forces.

The case of vertical trajectories is tackled similarly. Because $\theta(t) = 0$ deg, the corresponding $F_{\max}$ formula is the same as Eq. (11), but with the additional gravitational impulse term $n_{\mathrm{fill}}(m/m_{\mathrm{eff}})[gD_0/(V_i)^2]$ in the curly brackets. Going back to the case of the very strong pusher rocket where $\langle F_R\rangle \gg mg$, this result reduces to

$$F_{\max} \sim \left(\frac{\langle F_R\rangle + mg}{I_F^{if}}\right) \qquad \text{Vertical inflation} \tag{13}$$

Note that this easy-to-remember formula is very similar in form to one derived in the large-R_m and large-n_{fill} limit of $F_{\max}$, for parachutes dropped from fixed points [2].

V. Conclusions

The use of the MI theorem has once again brought a great deal of clarification and shown how parachute inflation data generated by freely falling payloads can be related to the data generated by motored payloads. As in [1–4], the theorem has also shown interesting limits for which the value of $F_{\max}$ is expressed in very simple terms.

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References

[1] Potvin, J., "Universality Considerations for Graphing Parachute Opening Shock Factor Versus Mass Ratio," *Journal of Aircraft*, Vol. 44, No. 2, 2007, pp. 528–538.

[2] Potvin, J., "Momentum-Impulse Balance and Parachute Inflation: Fixed-Point Drops," *Journal of Aircraft*, Vol. 44, No. 3, 2007, pp. 1026–1029.

[3] Potvin, J., "Momentum-Impulse Balance and Parachute Inflation: Clusters," *Journal of Aircraft*, Vol. 44, No. 2, 2007, pp. 687–691.

[4] Potvin, J., "Momentum-Impulse Balance and Parachute Inflation: Disreefing," *Journal of Aircraft*, Vol. 44, No. 2, 2007, pp. 691–694.

[5] Knacke, T. W., *Parachute Recovery Systems Design Manual*, Para Publishing, Santa Barbara, CA, 1992.

[6] Ewing, E. G., Bixby, H. W., and Knacke, T. W., "Recovery Systems Design Guide," pp. 254–257; also Rept. AFFL-TR-78-151, submitted to Air Force Flight Dynamics Laboratory, AF Wright Aeronautical Laboratories, Wright–Patterson Air Force Base, Dec. 1978 (unpublished).

[7] Wolf, D., "Opening Shock," AIAA Paper 99-1702, 1999.

[8] Sarpkaya, T., and Isaacson, M., *Mechanics of Wave Forces on Offshore Structures*, Van Nostrand Reinhold Co., New York, 1981.

[9] Iversen, H. W., and Balent, R., "A Correlating Modulus for Fluid Resistance in Accelerated Motions," *Journal of Applied Physics*, Vol. 22, No. 3, 1951, pp. 324–328.

[10] Potvin, J., "Accelerating Disk Drag: Theory and Applications," AIAA Paper 2002-3055, 2002.

[11] Cochran, B. C., White, B. R., and Macha, J. M., "Experimental Investigation of Added Mass During Parachute Deceleration—Preliminary Results", AIAA Paper 91-0853, 1991, pp. 171–180.

[12] Higuchi, H., Balligand, H., and Strickland, J. H., "Numerical and Experimental Investigations of the Flow Over a Disk Undergoing Unsteady Motion," *Journal of Fluids and Structures*, Vol. 10, 1996, pp. 705–719.

[13] Balligand, H., and Higuchi, H., "Wake of a Decelerating Disk," AIAA Paper 93-1218, May 1993.

[14] Balligand, H., and Higuchi, H., "Experimental Investigation of the Wake Behind a Solid Disk," Sandia National Laboratories Rept. SAND90-7083, Dec. 1993.

[15] Strickland, J. H., and Macha, M., "Preliminary Characterization of Parachute Wake Recontact," *Journal of Aircraft*, Vol. 27, No. 6, 1990, pp. 501–506.

[16] Spahr, H. R., and Wolf, D. F., "Theoretical Analysis of Wake-Induced Parachute Collapse," AIAA Paper 81-1922, 1981.

[17] Potvin, J., Peek, G., and Brocato, B., "New Model of Decelerating Bluff Body Drag," *Journal of Aircraft*, Vol. 40, No. 2, 2003, pp. 370–377.